COSMOLOGICAL & ASTROPHYSICAL SIMULATION

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• Uses

- Structures
- Information

• Projects

- Millenium, Eris, Virgo Consortium, Bolshoi
- Pleiades, Titan, Hyades

• Physics

- Cosmological Principle
- General Relativity
- N-Body Problem
- Hydrodynamics

Methods

- Brute Force
- Particle Mesh
- Tree Method
- Hybrids
- Initial Conditions

Software

- ChaNGa
- GADGET
- GRAPE
- RAMSES



STRUCTURES OF INTEREST

- In Planets
- Galaxies
- Stars
- CDM Halos
- IGM/ISM
- Big Bang



 Exotic objects (black holes, neutron stars, supernovae, gamma-ray bursts)

USES OF SIMULATIONS

- Explaining Observation
- Choosing Observation Targets
- Explaining Structure Formation
- Predicting Evolution of Structure

A basic scientific tool to test theories in cosmology is to evaluate their consequences for the observable parts of the universe. One piece of observational evidence is the distribution of matter, including galaxies and intergalactic gas, which are observed today. Light emitted from more distant matter must travel longer in order to reach Earth, meaning looking at distant objects is like looking further back in time. This means the evolution in time of the matter distribution in the Universe can be observed.



PROJECTS

Millennium Run (cosmology)

- 2e⁹ ly cube, 10¹⁰ particle (dark matter) N-body simulation
- Starts at emission of CMB to utilize inhomogenous matter distribution initial conditions
- 25 TB output data, > 1 month computing time
- Eris (galactic formation)
 - 6e⁷ particles, 13 billion years
 - Dark matter foundation & galactic gas
 - Feedback (UV background heating, supernovae blast wave enrichment, cooling)
 - 9 months computing time (1.6 million cpu hours)



PROJECTS

Bolshoi (cosmology)

- More accurate than Millennium run due to more accurate input (WMAP)
- (λCDM) Normal matter falls into gravitational wells produced by dark matter
- 10⁹ ly cube, 8.6*10⁹ particles, 6 million cpu hours

• Virgo Consortium

- Cosmology Machine at Durham
- Hubble Volume
- Nodes in Japan, UK, US, Germany, Canada, Netherlands
- Matter distribution, DM halos, galaxies & clusters, IGM, intracluster gas



VIRC

SUPERCOMPUTERS (PARALLELISM!)

• Pleiades

NAMES & COMPANY

- NASA Ames Research Center
- 126000 processors at 1.24 petaflops (19th)
- Titan
 - EVEREST visualization
 - Oak Ridge National Laboratory
- Hyades



- Supernovae, black holes, magnetic fields, planet formation, cosmology
- one of the largest repositories of astrophysical data outside of national facilities

COSMOLOGICAL PRINCIPLE

- Viewed on sufficiently large distance scales, there are no preferred directions or preferred places in the Universe
 - At very large scales, all parts of the universe look the same
 - Larger than supercluster scales (100-200 Mpc)
 - Homogenous (no special places)
 - Isotropic (no special directions)
- Corollary: The laws of physics are universal
- Implies that all parts of space are causally connected at some time in the past



GENERAL RELATIVITY

- Since gravity is the dominant force that governs the large scale dynamics of the universe, general relativity should be the basis for a model of the universe
- Friedmann metric describes a universe which is spatially homogenous and isotropic at each instant of time
 - This describes the shape and expansion of the universe
 - Positive, zero, or negative curvature



N-BODY PROBLEM

- Real matter does not need to be accounted for because dark matter is so much more abundant (CDMH)
- Collisionless (solely gravitation) dynamic system
- Non-relativistic fluid

http://nowykurier.com/toys/gravity/gravity.html



GOAL OF SIMULATION



HYDRODYNAMICS

 General relativity for large scale, hydrodynamics for structure formation

• Poisson-Vlasov System

 Phase space dark matter distribution function f(x,p,t)

$$\circ \frac{df}{dt} + \frac{p}{ma^2}\nabla f - m\nabla \Phi \frac{df}{dp} = 0$$

- $\nabla^2 \Phi(x,t) = 4\pi G a^2 [\rho(x,t) \overline{\rho}(t)]$
- Coupled



PARTICLE-PARTICLE/DIRECT SUM

- This method is the brute force way to simulate O(N²)
- Algorithm
 - Calculate force on each particle by every other particle using $\sum \frac{Gmm}{r^2}$
 - Integrate to find equations of motion and obtain updated velocities and positions (since we know the time step) of particles
 - Use a softening parameter such that r=(r²+ε²) to avoid divergence when things are very close (since they don't collide this is basically a finite radius of the particles)

PARTICLE MESH

- Mesh used to describe the field quantities & compute derivatives
- Re-write equations in Fourier space, use FFT
- Fast, but low resolution O(N+NlogN)
- Algorithm
 - Compute density at each grid point from the initial conditions
 - Solve the Poisson equation for the potential
 - Find the force at each point
 - Estimate the force on each particle by all the others using interpolation
 - Integrate equations of motion

DENSITY COMPUTATION

- The density at a point is given by
 - $\rho(x_{i,j,k}) = m_p M^3 \sum_{l=1}^{N_p} W(\delta x_l)$
 - Interpolation function W
 - Distance between l^{th} particle and particle in consideration δx_l
- Interpolation functions
 - Nearest Grid Point
 - Mass of particle is directly assigned to each point (discontinuity)
 - Density is mass of particle divided by volume of grid division
 - Cloud in Cell
 - Mass of particle is assigned to the 8 grids surrounding it inversely proportional to their distance away
 - Now the density is weighted and continuous
 - Triangular Shaped Cell
 - Same as CIC but uses more nodes for average

SOLVING POISSON EQUATION

- Transform potential function into Fourier space
 - $\Phi_k = G_k \delta_k$
 - Multiplication of (FT) density contrast and a suitable Green function.
- Differentiate the potential using finite difference methods to obtain force
- Interpolate the forces back to the particle positions using the same weighting scheme as before



 Instead of considering the forces on a body produced by every other body, allow a multipole expansion of many distant forces be accounted for by a single multipole force

Algorithm

- Root of tree contains the whole space, nodes of each root contain subdivisions of the root
- Space is recursively divided so that each leaf contains either 1 or 0 particles

 To calculate the force on a particle, traverse the tree, and if a corresponding node is far enough away, consider its multipole force, if not, continue traversing



HYBRID (TREEPM)

- Hybrid methods use Tree method to compute short range potential and Particle-Mesh to compute long range potential
- Solve long range with PM-method
- Solve short range with Tree method



INITIAL CONDITIONS

- Most important part of the effort!
- Gaussian random field of initial density fluctuations
- Considerations
 - Time step
 - Starting redshift
 - Accuracy of computations
 - N
 - Gravitational softening



SOFTWARE

• ChaNGa (Charm N-body Gravity Solver)

- Tree method for gravity
- Particle Mesh Method for electrostatic interaction

• GADGET (Galaxies with Dark matter and Gas interact)

- TreePM
- Zoom-in simulations

• GRAPE (Gravity Pipe)

- Direct force summation
- Computes gravitational force with a hardwire Plummer force law

RAMSES

- Fully threaded tree (cell contains references of neighbor cells so you don't have to traverse the whole tree to find these)
- Adaptive mesh refinement (grid size changes in certain region for better calculation in that region)



SOURCES

- http://wiki.tomabel.org/index.php?title=Vlasov-Poisson_System
- https://www.cfa.harvard.edu/~kenyon/pf/code/coag.htm
- https://webhome.phys.ethz.ch/~jguedes/eris.html
- http://iopscience.iop.org/0004-637X/478/1/13/fulltext/34284.text.html
- http://www.mpagarching.mpg.de/galform/virgo/millennium/
- http://news.ucsc.edu/2011/09/bolshoi-simulation.html
- http://www.astro.iag.usp.br/~xiiieaa/talks/springel-1.pdf
- arxiv.org/pdf/0801.1023v1.pdf
- http://icc.ub.edu/~liciaverde/IC/como.pdf
- http://mathworld.wolfram.com/FastFourierTransform.htm l
- http://arxiv.org/pdf/astro-ph/9911025v2.pdf